urwitz polynomial:

$$H(S) = \frac{N(S)}{D(S)}$$

* Conditions of Hurwitz polynomial:

1) All Coefficients are +ve
2) All Coefficients are exist.

ex 1:
Test whether the following polynomial is Hurwitz or not?

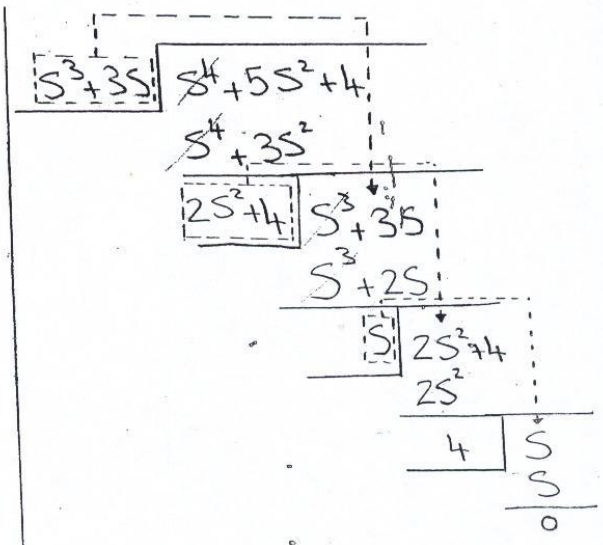
$$P(S) = S^4 + S^3 + 5S^2 + 3S + 4$$

$\Longrightarrow$ even part $m(S) = S^4 + 5S^2 + 4$

$\Longrightarrow$ odd part $n(S) = S^3 + 3S$

$\because$ degree of $m(S)$ > degree of $n(S)$

$\therefore$ $n(S) \overline{)\, m(S)}$



All quotient are +ve

$\therefore$ It is Hurwitz

$q_1 = S$ +ve

$q_2 = \frac{S}{2}$ +ve

$q_3 = 2S$ +ve

$q_4 = \frac{S}{4}$ +ve

Test Hurwitz or not?

$$f(s) = s^4 + s^3 + 2s^2 + 3s + 2$$

- even part $m(s) = s^4 + 2s^2 + 2$
- odd part $n(s) = s^3 + 3s$

$s^3+3s \;|\; s^4+2s^2+2$

s^4+3s^2

$-s^2+2 \;|\; s^3+3s$

$q_1 = s$ (+ve)

$q_2 = -s$ (−ve)

∴ It is not Hurwitz.

---

ex3:

$$f(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$$

⇒ even part $m(s) = 2s^6 + s^4 + 8s^2 + 4$

⇒ odd part $n(s) = s^7 + 2s^5 + 4s^3 + 8s$

$2s^6+s^4+8s^2+4 \;|\; s^7+2s^5+4s^3+8s$

$s^7+\frac{1}{2}s^5+4s^3+2s$

$\frac{3}{2}s^5+6s \;|\; 2s^6+s^4+8s^2+4$

$2s^6 \quad +8s^2$

$s^4+4 \;|\; \frac{3}{2}s^5+6s$

$\frac{3}{2}s^5+6s$

$0+0$

$q_1 = \frac{1}{2}s$ (+ve)

$q_2 = \frac{4}{3}s$ (+ve)

$q_3 = \frac{3}{2}s$ (+ve)

There is only 3 quotient only ∴ There is Common factor

$$
\begin{array}{r|l}
 & s^3+2s^2+2s+1 \\
\hline
s^4+4 & s^7+2s^6+2s^5+s^4+4s^3+8s^2+8s+4 \\
 & s^7 \qquad\qquad +4s^3 \\
\hline
 & 2s^6+2s^5+s^4+8s^2+8s+4 \\
 & 2s^6 \qquad\qquad +8s^2 \\
\hline
 & 2s^5+s^4+8s+4 \\
 & 2s^5 \qquad +8s \\
\hline
 & s^4+4 \\
 & s^4+4 \\
\hline
 & 0+0
\end{array}
$$

$\therefore f(s) = (s^4+4)(s^3+2s^2+2s+1)$

but $s^4+4=0$

$s^2 = \pm 2j$

$\because$ there is multipoles at j-axis

$\therefore$ not Hurwitz

Test Hurwitz or not?

* $F(S) = S^3 + 2S^2 + 3S + 6$

$m(S) = 2S^2 + 6$

$n(S) = S^3 + 3S$

$q_1 \rightarrow \frac{S}{2}$ (+ve)

$$2S^2+6 \;\overline{)\; S^3+3S}$$

$S^3 + 3S$

0

* there must be 3 quotients, since there is only 1 q. ∴ there must be a common factor

$$2S^2+6 \;\overline{)\; S^3+2S^2+3S+6} \quad \text{quotient: } \tfrac{1}{2}S+1$$

$S^3 + 3S$

$2S^2 + 6$

$2S^2 + 6$

0

$F(S) = (2S^2+6)(\tfrac{1}{2}S+1)$

$2S^2 + 6 = 0$

$\therefore S = \pm j\sqrt{3}$

$\tfrac{1}{2}S + 1 = 0$

$S = -2$

∴ poles are in LHP & has a single pole at j-axis

∴ it is Hurwitz

.est Hurwitz. or not?

$$F(S) = \frac{S^2+2S+1}{S^3+2S^2+2S+40}$$

$P(S) = N(S) + D(S)$

$P(S) = S^3+3S^2+4S+41$

$m(S) = 3S^2+41$

$n(S) = S^3+4S$

$q_1 \rightarrow \boxed{\frac{S}{3}}$ +ve

$q_2 \rightarrow \boxed{\frac{-9S}{29}}$ -ve

$3S^2+41 \;\big|\; S^3+4S$

$S^3 + \frac{41}{3}S$

$\frac{-29}{3}S \;\big|\; 3S^2+41$

∴ It is non-Hurwitz

Positive real function (PRF)

[1]

⇒ To be PRF the following requirements are fullfield:

1) $m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0 \quad \forall \omega$ ⟵ الشرط لازم يتحقق لكل قيم ω

m_1: even part of $N(s)$ $\quad n_1$: odd part of $N(s)$

n_2: even part of $D(s)$ $\quad n_2$: odd part of $D(s)$

2) All Coeff. are real & +ve

3) all poles lie in LHP. → must be Hurwitz

$$F(s) = \frac{N(s)}{D(s)}$$

$$P(s) = N(s) + D(s)$$

⇒ Properties of PRF:

1) if $F(s)$ is PRF ∴ $\frac{1}{F(s)}$ is also PRF "if $Z(s)$ is PRF ∴ $Y(s)$ is PRF"

2) The sum of PRFs is PRF but the difference is not necessary.

if $Z_1(s)$ & $Z_2(s)$ are PRF

$F(s) = Z_1(s) + Z_2(s)$ ∴ $F(s)$ is PRF

$P(s) = Z_1(s) - Z_2(s)$ ∴ $P(s)$ is not necessary PRF

Test PRF or not & give reasons. [2]

1:

$$F(S)=\frac{S^2+5S}{S^4+2S^2+1}=\frac{}{(S^2+1)^2}$$ ↳ not PRF bec. it has multi poles at jω axis

2:

$$Y(S)=\frac{9S+2}{S+2}$$

$S+2=0$

$\therefore S=-2$

has no poles at RHP

$m_1(S)=2$, $n_1(S)=9S$

$m_2(S)=2$, $n_2(S)=S$

$$m_1m_2-n_1n_2\Big|_{S=j\omega}\geq 0 .$$

$$4-9S^2\Big|_{S=j\omega}\geq 0$$

$4+9\omega^2\geq 0$ for all ω ($\forall\omega$)

$\therefore$ F(S) is PRF.

[3]

x3:

$$F(S)=\frac{2S^3+3S^2+3S+3}{S^2+1}$$

$P(S)=2S^3+4S^2+3S+4$

$m(S)=4S^2+4$

$n(S)=2S^3+3S$

$4S^2+4 \;\overline{)\; 2S^3+3S}$

$2S^3+2S$

$S \;\overline{)\; 4S^2+4}$

$4S^2$

$4 \;\overline{)\; S}$

S

0

$+ve\ q_1=\frac{1}{2}S$

$+ve\ q_2=4S$

$+ve\ q_3=\frac{1}{4}S$

∴ Herwitz

$n_1=3S^2+3 \quad , n_1=2S^3+3S$

$n_2=S^2+1 \quad , n_2=0$

$m_1m_2-n_1n_2\big|_{S=j\omega}\geq 0 \ \forall\omega$

$3S^4+6S^2+3\big|_{S=j\omega}\geq 0 \ \forall\omega$

$3\omega^4-6\omega^2+3\geq 0 \ \forall\omega$

∴ $F(S)$ is PRF

x4: Determine the Condition for biquadratic function to be PRF (41

$$F(S) = \frac{S^2 + a_1 S + a_0}{S^2 + b_1 S + b_0}$$

a_0, a_1, b_0, b_1 are +ve & real

$$m_1 = S^2 + a_0 \quad , n_1 = a_1 S$$
$$m_2 = S^2 + b_0 \quad , n_2 = b_1 S$$

$$m_1 m_2 - n_1 n_2 \geq 0$$

$$(S^2 + a_0)(S^2 + b_0) - a_1 b_1 S^2 \geq 0$$

$$S^4 + (a_0 + b_0) S^2 + a_0 b_0 - a_1 b_1 S^2 \geq 0$$

$$S^4 + (a_0 + b_0 - a_1 b_1) S^2 + a_0 b_0 \geq 0$$

$$\omega^4 - (a_0 + b_0 - a_1 b_1) \omega^2 + a_0 b_0 \geq 0$$

assume $\omega^4 = x^2$

$\therefore \omega^2 = x$

$$x^2 - (a_0 + b_0 - a_1 b_1) x + a_0 b_0 \geq 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{a_0 + b_0 - a_1 b_1 \pm \sqrt{[-(a_0 + b_0 - a_1 b_1)]^2 - 4 a_0 b_0}}{2}$$

$$(a_0 + b_0 - a_1 b_1)^2 > 4 a_0 b_0$$

$$a_0 + b_0 - a_1 b_1 > 2\sqrt{a_0 b_0}$$

$$a_0 + b_0 - 2\sqrt{a_0 b_0} > a_1 b_1$$

$$(\sqrt{a_0} - \sqrt{b_0})^2 > a_1 b_1$$

$b^2 \geq 4ac$ → Condition to make sure that ω is real

<5>

$$F(s) = \frac{s^2+2s+a}{s^3+3s+2}$$

Find a to be PRF

$n_1 = s^2 + a \quad , n_1 = 2s$

$m_2 = s^2 + 2 \quad , n_2 = 3s$

$m_1 m_2 - n_1 n_2 \geq 0$

$(s^2+a)(s^2+2) - 6s^2 \geq 0$

$s^4 + (a-4)s^2 + 2a \geq 0$

$s = j\omega$

$\omega^4 - (a-4)\omega^2 + 2a \geq 0$

→ Real, +ve

$b^2 \geq 4ac$

$[-(a-4)]^2 \geq 4 \times 2a$

$(a-4)^2 \geq 8a$

$a - 4 \geq 2\sqrt{2a}$

x 6:

$$F(S) = \frac{S+a}{S^2+bS+c}$$

a, b, c are +ve & real

$m_1 = a \quad , n_1 = S$

$m_2 = S^2 + c \quad , n_2 = bS$

$$m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0$$

$$aS^2 + ac - bS^2 \Big|_{s=j\omega} \geq 0$$

$$(a-b)S^2 + ac \Big|_{s=j\omega} \geq 0$$

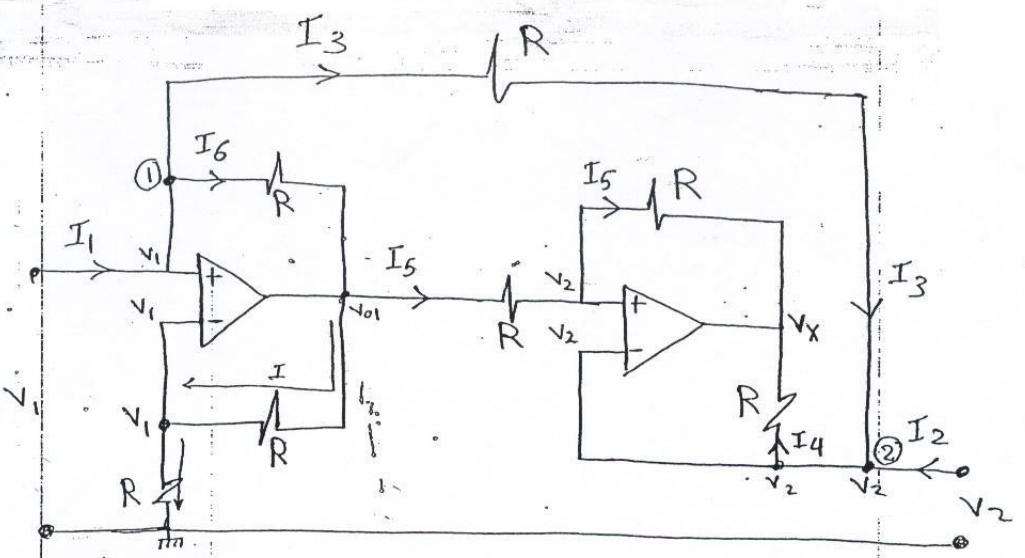
$$-(a-b)\omega^2 + ac \geq 0$$

$$(b-a)\omega^2 + ac \geq 0$$

$b - a > 0 \quad \therefore b > a$

$c > 0$

(i)



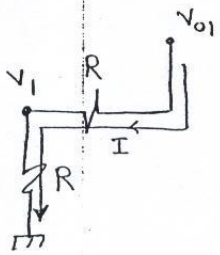
apply KCL at node (2) :-

$I_4 = I_2 + I_3$

$I_2 = I_4 - I_3 = \frac{V_2 - V_X}{R} - \frac{V_1 - V_2}{R}$

$$\boxed{I_2 = \frac{V_2 - V_X}{R} - \frac{V_1 - V_2}{R}} \quad (1)$$

$$\boxed{I_5 = \frac{V_{01} - V_2}{R} = \frac{V_2 - V_X}{R}} \quad (2)$$



$\frac{V_{01} - V_1}{R} = \frac{V_1}{R}$

$$\boxed{2V_1 = V_{01}} \quad (3)$$

Substitute from (3) in (2) :-

$2V_1 - V_2 = V_2 - V_X$

$$\boxed{V_X = 2V_2 - 2V_1} \quad (4)$$

Substitute from (4) in (1) :-

$I_2 = \frac{V_2}{R} - \frac{2V_2}{R} + \frac{2V_1}{R} - \frac{V_1}{R} + \frac{V_2}{R}$

$\therefore I_2 = \frac{V_1}{R}$

$$\boxed{V_1 = I_2 R}$$

apply KCL at node (1) :-

$I_1 = I_6 + I_3$

$I_1 = \frac{V_1 - V_{01}}{R} + \frac{V_1 - V_2}{R}$

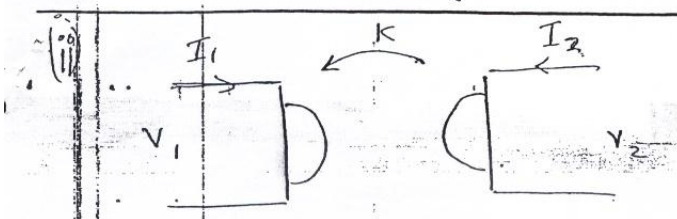
$\therefore \boxed{2V_1 = V_{01}}$

$\therefore I_1 = \frac{V_1}{R} - \frac{2V_1}{R} + \frac{V_1}{R} - \frac{V_2}{R}$

$$\boxed{I_1 = \frac{-V_2}{R}}$$

$\therefore T = \begin{bmatrix} 0 & -R \\ -1/R & 0 \end{bmatrix}$

this circuit realize a gyrator



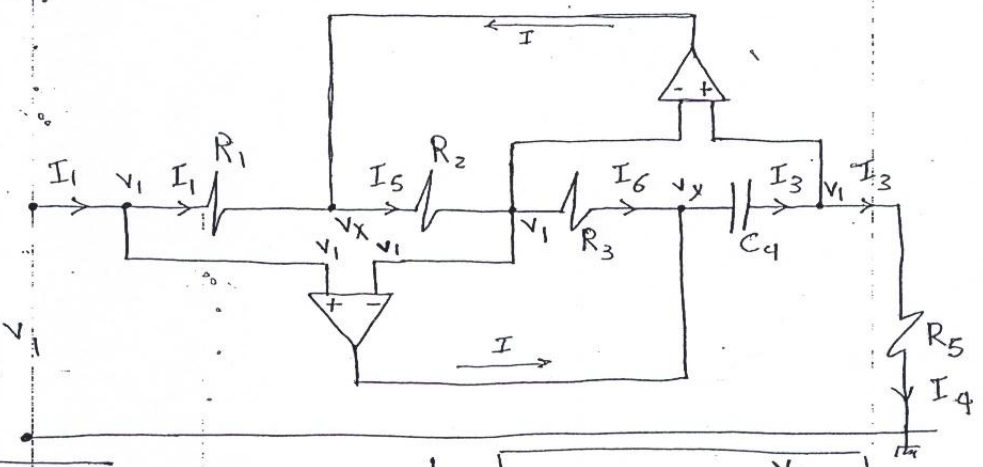
$$V_1 = I_2 K$$

$$V_2 = -I_1 K$$

$$I_1 = -\frac{1}{K} V_2$$

$$T = \begin{bmatrix} 0 & -K \\ -1/K & 0 \end{bmatrix}$$

2



$$I_1 = \frac{V_1 - V_x}{R_1} \quad (1)$$

$$I_5 = I_6$$

$$\therefore \frac{V_x - V_1}{R_2} = \frac{V_1 - V_y}{R_3} \quad (2)$$

$$I_3 = \frac{V_y - V_1}{1/SC_4} = \frac{V_1}{R_5}$$

$$\therefore (V_y - V_1) S C_4 = \frac{V_1}{R_5}$$

$$V_y - V_1 = \frac{V_1}{S C_4 R_5}$$

$$\therefore V_1 - V_y = -\frac{V_1}{S C_4 R_5} \quad (3)$$

from (3) in (2) :-

$$\frac{V_x - V_1}{R_2} = -\frac{V_1}{S C_4 R_5 R_3}$$

$$\therefore V_x = V_1 - \frac{V_1 R_2}{S C_4 R_5 R_3} \quad (4)$$

from (4) in (1) :-

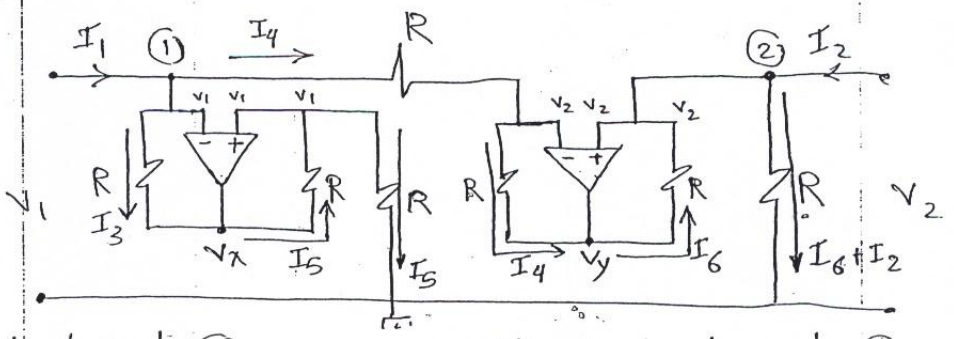
$$I_1 = \frac{V_1}{R_1} - \frac{V_1}{R_1} + \frac{V_1 R_2}{S C_4 R_5 R_3 R_1}$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{S C_4 R_5 R_3 R_1}{R_2} = SL$$

where :- $\boxed{L = \frac{C_4 R_5 R_3 R_1}{R_2}}$

this circuit is equivalent to a grounded inductor.

4)



apply kcl at node (1) :-

$I_1 = I_4 + I_3$

$\boxed{I_1 = \frac{V_1 - V_x}{R} + \frac{V_1 - V_2}{R}}$ (1)

$I_5 = \frac{V_x - V_1}{R} = \frac{V_1}{R}$

$\therefore \boxed{2V_1 = V_x}$ (2)

from (2) in (1) :-

$I_1 = \frac{V_1}{R} - \frac{2V_1}{R} + \frac{V_1}{R} - \frac{V_2}{R}$

$\therefore \boxed{I_1 = -\frac{V_2}{R}}$

apply kcl at node (2) :-

$I_6 + I_2 = \frac{V_2}{R}$

$\therefore I_2 + \frac{V_y - V_2}{R} = \frac{V_2}{R}$

$\therefore \boxed{I_2 + \frac{V_y}{R} = \frac{2V_2}{R}}$ (3)

$I_4 = \frac{V_1 - V_2}{R} = \frac{V_2 - V_y}{R}$

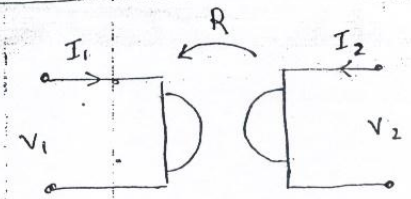
$\therefore \boxed{V_y = 2V_2 - V_1}$ (4)

from (4) in (3) :-

$I_2 + \frac{2V_2}{R} - \frac{V_1}{R} = \frac{2V_2}{R}$

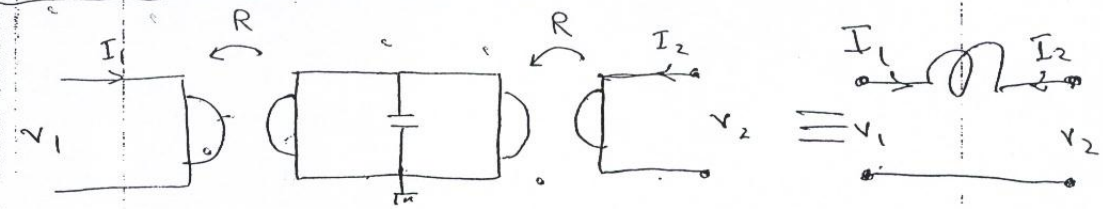
$I_2 = \frac{V_1}{R} \Rightarrow \boxed{I_2 R = V_1}$

this circuit realize a gyrator :-



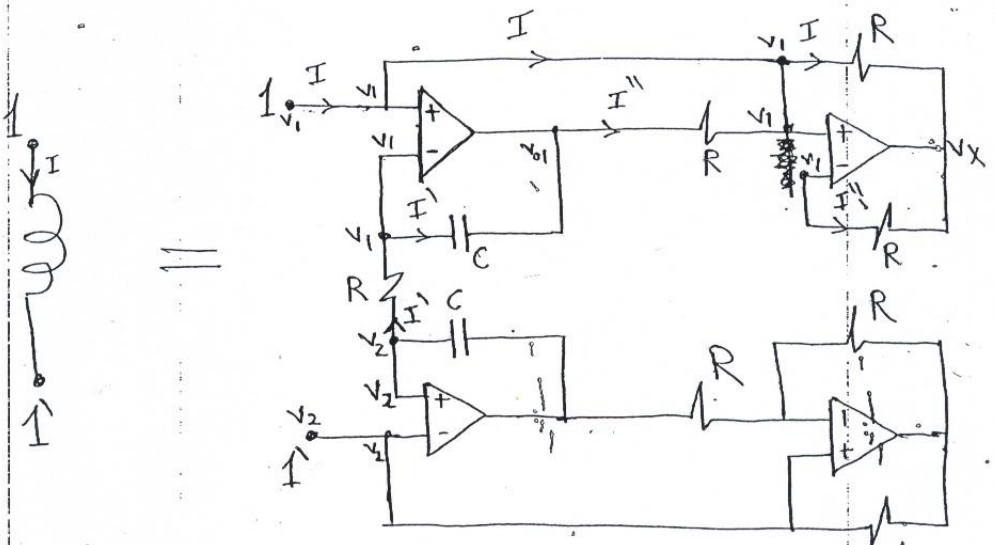
$$T = \begin{bmatrix} 0 & -R \\ -1/R & 0 \end{bmatrix}$$

we can simulate a floating inductor :-



the total number of element are needed to simulate the floating inductor = 4 opamp + 14R + 1C = 19

12



$$\frac{V_1 - V_X}{R} = I$$

$$\frac{V_1 - V_X}{R} = I''$$

$$\therefore \boxed{I = I''}$$

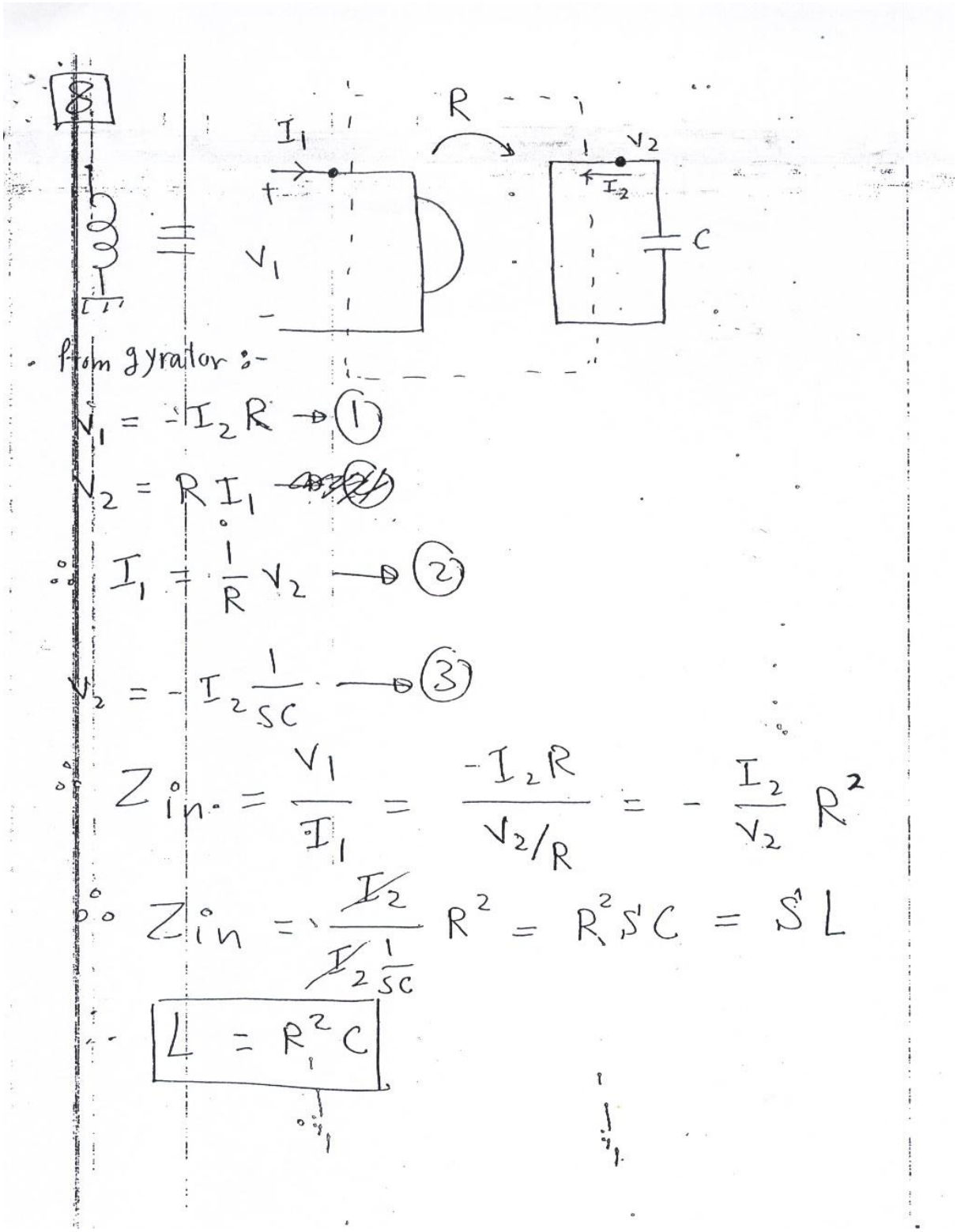
$$I' = \frac{V_2 - V_1}{R} = (V_1 - V_{01})\,SC$$

$$I'' = \frac{V_{01} - V_1}{R} = I$$

$$\therefore \frac{V_1 - V_{01}}{R} = \frac{V_2 - V_1}{R^2 SC} = -I$$

$$\therefore \boxed{\frac{V_1 - V_2}{R^2 SC} = I}$$

$$\therefore \frac{V_1 - V_2}{I} = R^2 SC = SL$$



- From gyrator :-

$$V_1 = -I_2 R \rightarrow (1)$$

$$V_2 = R I_1$$

$$I_1 = \frac{1}{R} V_2 \rightarrow (2)$$

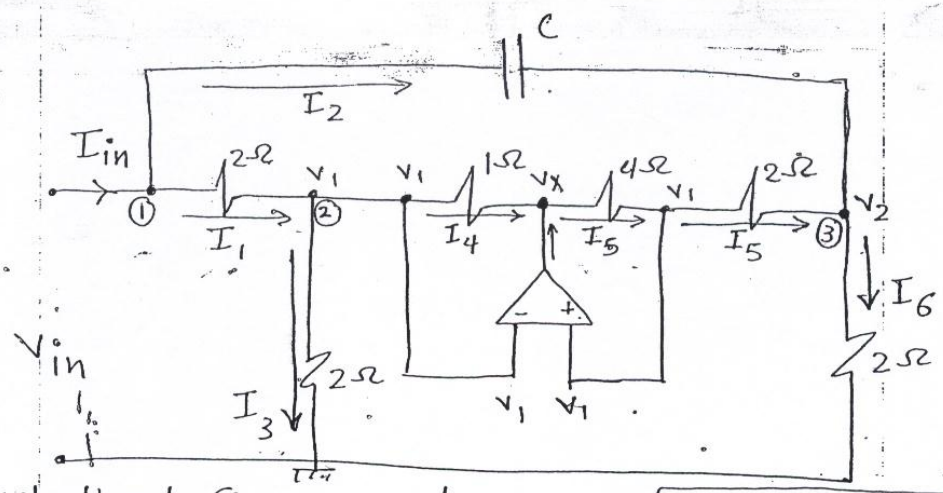
$$V_2 = -I_2 \frac{1}{SC} \rightarrow (3)$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{-I_2 R}{V_2/R} = -\frac{I_2}{V_2} R^2$$

$$\therefore Z_{in} = \frac{I_2}{I_2 \frac{1}{SC}} R^2 = R^2 SC = SL$$

$$\boxed{L = R^2 C}$$

B



apply KCL at node ① :-

$$I_{in} = I_1 + I_2 = \frac{V_{in} - V_1}{2} + \frac{V_{in} - V_2}{1/SC}$$

$$= \frac{1}{2} V_{in} - \frac{1}{2} V_1 + V_{in} SC - V_2 SC$$

$$\therefore \boxed{I_{in} = V_{in}\left[\frac{1}{2} + SC\right] - \frac{1}{2} V_1 - V_2 SC} \quad (1)$$

apply KCL at node ② :-

$$I_1 = I_3 + I_4 = \frac{V_1}{2} + \frac{V_1 - V_x}{1}$$

$$\therefore I_1 = V_1\left[\frac{1}{2} + 1\right] - V_x = \frac{3}{2} V_1 - V_x$$

$$\boxed{I_1 = \frac{3}{2} V_1 - V_x} \quad (2)$$

$$I_5 = \frac{V_x - V_1}{4} = \frac{V_1 - V_2}{2}$$

$$\frac{V_x}{4} - \frac{V_1}{4} = \frac{V_1}{2} - \frac{V_2}{2}$$

$$\therefore \boxed{V_x = 3V_1 - 2V_2} \quad (3)$$

from (3) in (2) :-

$$I_1 = 2V_2 - \frac{3}{2} V_1$$

We put $\boxed{I_1 = \frac{V_{in} - V_1}{2}}$

$$\therefore \frac{V_{in} - V_1}{2} = 2V_2 - \frac{3}{2} V_1$$

$$\therefore \boxed{V_{in} = 4V_2 - 2V_1} \quad (4)$$

apply KCL at node (B) :-

$$I_2 + I_5 = I_6$$

$$\frac{V_{in} - V_2}{1/SC} + \frac{V_1 - V_2}{2} = \frac{V_2}{2}$$

$$V_{in} SC - V_2 SC + \frac{1}{2} V_1 - \frac{1}{2} V_2 = \frac{1}{2} V_2$$

$$\therefore V_{in} SC = V_2\left[1 + SC\right] + \frac{1}{2} V_1$$

$$\boxed{V_{in} = V_2\left[\frac{1}{SC} + 1\right] - \frac{1}{2SC} V_1} \quad (5)$$

we equating equation (4) and (5) :-

$$4V_2 - 2V_1 = \frac{1}{SC} V_2 + V_2 - \frac{1}{2SC} V_1$$

$$2V_1 - \frac{1}{2SC} V_1 = 3V_2 - \frac{1}{SC} V_2$$

$$4SC V_1 - V_1 = 6SC V_2 - 2V_2$$

$$V_1\left[4SC - 1\right] = V_2\left[6SC - 2\right]$$

$$\boxed{V_2 = V_1 \frac{4sC - 1}{6sC - 2}} \quad (6)$$

From (6) in (4) :-

$$V_{in} = V_1 \left[\frac{16sC - 4}{6sC - 2} + 2 \right]$$

$$V_{in} = V_1 \left[\frac{16sC - 4 + 12sC - 4}{6sC - 2} \right]$$

$$V_{in} = V_1 \left[\frac{4sC}{6sC - 2} \right]$$

$$\boxed{V_1 = V_{in} \left[\frac{6sC - 2}{4sC} \right] = V_{in} \left[\frac{3}{2} - \frac{2}{4sC} \right]} \quad (7)$$

From (7) in (1) :-

$$I_{in} = V_{in} \left[\frac{1}{2} + sC \right] - \frac{1}{2} V_{in} \left[\frac{3}{2} - \frac{2}{4sC} \right] - sC V_{in} \left[\frac{3}{2} - \frac{2}{4sC} \right] \left[\frac{4sC - 1}{6sC - 2} \right]$$

$$= V_{in} \left[\frac{1}{2} + sC \right] - V_{in} \left[\frac{3}{4} - \frac{1}{4sC} \right] - V_{in} \left[\frac{3}{2} sC - \frac{1}{2} \right] \left[\frac{4sC - 1}{6sC - 2} \right]$$

$$\frac{I_{in}}{V_{in}} = -\frac{1}{4} + sC + \frac{1}{4sC} - \left[\frac{12s^2C^2 - 7sC + 1}{12sC - 4} \right]$$

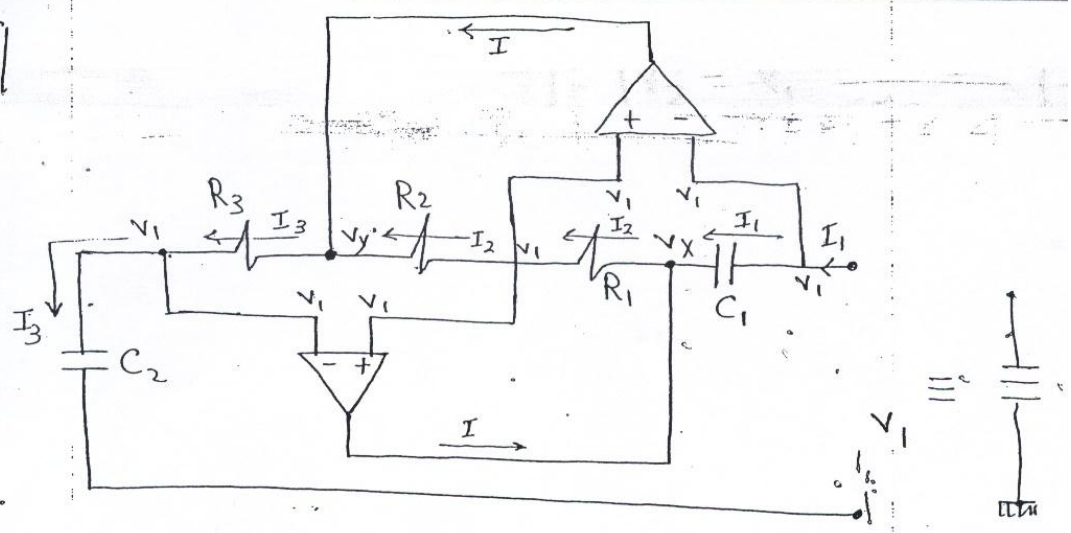
$$= -\frac{1}{4} + sC + \frac{1}{4sC} - \left[\frac{(4sC - 1)(3sC - 1)}{4(3sC - 1)} \right]$$

$$= -\frac{1}{4} + sC + \frac{1}{4sC} - sC + \frac{1}{4} = \frac{1}{4sC}$$

$$\therefore \frac{V_{in}}{I_{in}} = Z_{in} = 4sC = sL$$

$\therefore$ where $\boxed{L = 4C}$ #

4



$$I_1 = \frac{V_1 - V_x}{1/sC_1} = (V_1 - V_x)\,sC_1 \longrightarrow (1)$$

$$I_2 = \frac{V_x - V_1}{R_1} = \frac{V_1 - V_y}{R_2}$$

$$\therefore \frac{V_1 - V_x}{R_1} = \frac{V_y - V_1}{R_2} \longrightarrow (2)$$

From (2) in (1) :-

$$\frac{I_1}{sC_1R_1} = \frac{V_y - V_1}{R_2}$$

$$\therefore V_y - V_1 = \frac{R_2 I_1}{sC_1R_1} \longrightarrow (3)$$

$$I_3 = \frac{V_y - V_1}{R_3} = \frac{V_1}{1/sC_2} = V_1 sC_2 \longrightarrow (4)$$

∴ divide equation (3) by R_3 :-

$$\frac{V_y - V_1}{R_3} = \frac{R_2 I_1}{sC_1R_1R_3} \longrightarrow (5)$$

From (4) in (5) :-

$$\therefore V_1 sC_2 = \frac{R_2 I_1}{sC_1R_1R_3} \longrightarrow \boxed{\frac{V_1}{I_1} = Z_{in} = \frac{R_2}{s^2C_2C_1R_1R_3}} \rightarrow \text{FDNR}$$